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A Model for Optimal Base Stock System for Patient Customers With Reference To Distribution of Lead Time Having a Parametric Change

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Abstract- A new type of inventory model using the so called "Setting the Clock Back to Zero" (SCBZ) property is analyzed in this paper. Base Stock system for patient customers is a special type of ordering mechanism in inventory control theory. The inventory process begins with an initial inventory of B units. Whenever a customer's order for 'r' units is received, an inventory replenishment order of 'r' units is placed. Replenishment orders are fulfilled after the lead time L. If the demand exceeds the stock on hand, then assume that the customer will not cancel the order but await the arrival of sufficient stock. Here we assume that L is a random variable and it satisfies the SCBZ property and so the distribution undergoes a parametric change after the truncation point. Assuming that the truncation point is a random variable which has the mixed exponential distribution, the optimal value of Base Stock is derived.

Index Terms- Base Stock, Lead time, SCBZ property and Truncation point.

1. INTRODUCTION

In inventory control theory, the very famous Base Stock system for patient customers is an important model and these models have been discussed by many authors. Base Stock system for patient customers in inventory control theory is an interesting method of ordering mechanism. The inventory initially begins with 'B' number of units. Whenever a customer order for 'r' units is received, an inventory replenishment order for 'r' units is placed immediately. Replenishment orders are filled after the lead time 'L'. The customer's demand is met with as far as possible from the supply on hand. If the total unfilled customer's demand surplus the supply on hand, then assume that the customer will not cancel the orders but the customers wait till their requirement is fulfilled. The sum of inventory on hand and order placed is constant in time and equal to 'B', called as Base Stock. The detailed study of the Base Stock system for patient customers has been discussed by Gaver[1].
The very basic model had been discussed by Hanssman[2]. Ramanarayanan et.al[5] have discussed the model in which the lead time was assumed to be a random variable. Sachithanantham et. al[6] have discussed the modified version of the Base Stock model in which the lead time was assumed to be a random variable and which satisfies the so called Setting the Clock Back to Zero (SCBZ) property. In the sense that after the truncation point the lead time takes parametric change. The change point known as the truncation point and is also assumed to be a random variable and based on these assumptions, optimal Base Stock has been obtained. The concept of SCBZ property was basically discussed by RajaRao and Talwalker[4] . Henry et. al[3] have discussed the Base Stock model with the assumption that the lead time random variable has the SCBZ property and assuming that the truncation point itself a random variable which follows exponential distribution. Based on these assumptions the optimal Base Stock has been obtained.
In this paper, it is assumed that the lead time random variable follows exponential distribution which satisfies the SCBZ property and the change point is itself a random variable which has the mixed exponential distribution. Under this assumption, the optimal Base Stock is obtained.

2. PROBLEM FORMATION

Consider the Base Stock system, in which let X be the amount of demand during the lead time L. If f(x) is the probability density function of demand, then the Expected cost per unit time of overages and shortages attributed to the inventory on ground is

$$E(C) = h\int_0^B (B-X)\,f(x)dx + d\int_B^\infty (X-B)\,f(x)dx$$

Therefore the optimal Base Stock $\hat{B}$, can be obtained by using $\frac{dE(C)}{dB} = 0$. It follows that $F(\hat{B}) = \frac{d}{h+d}$ (2.1)

Here F(x) is distribution of f(x).
If there are N demand epochs during L and N is a random variable then the probability that there are exactly 'n' demand during L is

$$P[N = n/L] = G_n(L) - G_{n+1}(L) \text{ [from the renewal theory arguments]}$$

Let X be the total demand during L, say X = $X_1+X_2+ \ldots..+X_N$, then the total demand is at the most x, during L, is given by

$$P[X \le x] = \sum_{n=0}^{\infty} P[X_1 + X_2 + \cdots + X_n \le x/N = n] \, . \, P(N = n/L)$$

$$= \sum_{n=0}^{\infty} [G_n(L) - G_{n+1}(L)] \, . \, F_n(x)$$

Therefore, the expected cost is,

$$E(C) = h \int_0^B (B - X) \, dP(X \le x) + d \int_B^{\infty} (X - B) \, dP(X \le x)$$

Thus the optimal Base Stock is given by

$$F(\hat{B}) = \frac{d}{h+d} = P[X \le B]$$

Hence, $\frac{d}{h+d} = \sum_{n=0}^{\infty} [G_n(L) - G_{n+1}(L)] \, . \, F_n(B)$ (2.2)

In the above, we assume that the lead time L be a continuous random variable with probability density function K(.). For the sake of convenience, take L= Y.

Therefore the optimal Base Stock is given by

$$\sum_{n=1}^{\infty} F_n(B) \int_0^{\infty} [G_n(L) - G_{n+1}(L)] k(y) dy = \frac{d}{h + d} \tag{2.3}$$

2.1. NOTATIONS

B	:	The Base Stock level.
L	:	A random variable denoting the lead time with the pdf is k(.).
U	:	Random variable denoting the inter-arrival times between successive demands during the lead time with Pdf g(.) and cdf G(.).
λ	:	Parameter of inter arrival time distribution.
$G_n(.)$	:	The n^{th} convolution of G(.).
X_i	:	A random variable denoting the magnitude of demand at the i^{th} demand epoch with pdf f(.) and F(.) is the cdf and X_i ~ exp (μ), i = 1,2,3,…..n.
µ	:	Parameter of demand distribution.
$F_k(.)$	:	The k^{th} convolution of F(.).
h	:	Inventory holding cost / unit / time.
d	:	Shortage cost / unit / time.

2.2. DEFINITIONS AND ASSUMPTIONS

2.2.2. Definition

Setting the Clock Back to Zero property (SCBZ property)

The special property known as Setting the Clock Back to Zero property (SCBZ property) is due to RajaRao. B and Talwaker[4]. The family of life distributions { $f(x, \theta)$, $x \ge 0$, $\theta \in \Omega$ } is said to have be ' SCBZ property' is the form of $f(x, \theta)$ remains unchanged except for value of the parameter.

i.e., $f(x, \theta) \rightarrow f(x, \theta^*)$ where $\theta^* \in \Omega$

Under the following three operations
(i) Truncating original distribution of some point $X_0 \ge 0$
(ii) Considering the observable distribution for life time $X \ge X_0$ and
(iii) Changing the origin by means of the transformation given by $X_1 = -X_0$, $X_1 \ge 0$, where X_0 is a Truncation point.

2.2.2. Assumptions

1. The lead time is continuous random variable and its probability density function undergoes a parametric change(SCBZ property) after the truncation point. Here the pdf of lead time random variable is exponential which takes the parametric change.

$$\text{i.e.,}\qquad f(x) = \begin{cases} f(x,\theta) = \theta e^{-\theta x} & ,if\ X \le X_0 \\ f(x,\theta^*) = \theta^* e^{-\theta^* x}\ e^{x_0(\theta^*-\theta)} & ,if\ X > X_0 \end{cases}$$

Where X_0 is a Truncation point.

2. The truncation point is a random variable, which follows the mixed exponential distribution with parameter τ and δ.

Here, $f(x) = f(\,x,\theta\,)P(X \le X_0) + f(\,x,\theta^*\,)\ P(X > X_0)$

3. The inter-arrival times between successive demand epochs are i.i.d random variables and are assumed to be followed as exponential with parameter λ.

3. MAIN RESULTS

In this section, we discuss the approach to determine optimal Base Stock inventory under the assumptions that the lead time random variable follows exponential distribution, which satisfies the so called SCBZ property and it is also assumed that the truncation point itself a random variable, which has the mixed exponential distribution with parameters τ and δ. In this model, it is assumed that the lead time random variable follows an exponential distribution, which satisfies the so called SCBZ property and it is also assumed that the truncation point itself a random variable which has the mixed exponential distribution with parameter τ and δ. The SCBZ property, basically is discussed by RajaRao Talwaker[4].Under the said assumptions the optimal Base Stock is given in equation (2.3) as

$$\frac{d}{h+d} = \sum_{n=1}^{\infty} F_n(B)\left\{\int_0^{\infty} [G_n(Y) - G_{n+1}(Y)]K(Y)dy\right\} \tag{3.3}$$

$$K(y) = \begin{cases} \theta e^{-\theta y} & ,if\ Y \le y_0 \\ \theta^* e^{-\theta^* y}\ e^{y_0(\theta^*-\theta)} & ,if\ Y > y_0 \end{cases}$$

Here the truncation point Y_0 is a random variable which follows mixed exponential distribution and thus,

$$K(y) = K(\,y,\theta\,)P(Y \le y_0) + K(\,y,\theta^*\,)\ P(Y > y_0)$$

$$K(y) = \theta e^{-\theta y}P(Y \le y_0) + \theta^* e^{-\theta^* y}\ e^{Y_0(\theta^*-\theta)}P(Y > y_0)$$

$$f(y_0) = \beta\tau e^{-\tau y_0} + (1-\beta)\delta e^{-\delta y_0}$$

$$\begin{aligned} \therefore P(Y \le y_0) &= P(y_0 \ge Y) \\ &= \int_y^{\infty} f(y_0)dy_0 \\ &= \beta e^{-\tau t} + (1-\beta)e^{-\delta t} \end{aligned}$$

$$\begin{aligned} \therefore K(y) &= \theta\, e^{-\theta y}\left(\beta e^{-\tau y} + (1-\beta)e^{-\delta y}\right) + \int_0^y \theta^* e^{-\theta^* y} e^{y_0(\theta^*-\theta)}\left(\beta\,\tau\, e^{-\tau y_0} + (1-\beta)\delta e^{-\delta y_0}\right)dy_0 \\ &= \theta\, e^{-\theta y}\left(\beta e^{-\tau y} + (1-\beta)e^{-\delta y}\right) + \theta^* e^{-\theta^* y}\int_0^y e^{y_0(\theta^*-\theta)}e^{-\tau y_0}dy_0 \\ &\quad + \theta^* e^{-\theta^* y}(1-\beta)\delta\int_0^y e^{y_0(\theta^*-\theta)}e^{-\delta y_0}dy_0 \\ &= A + \theta^* e^{-\theta^* y}\tau\beta\int_0^y e^{y_0(\theta^*-\theta-\tau)}dy_0 + \theta^* e^{-\theta^* y}(1-\beta)\delta\int_0^y e^{y_0(\theta^*-\theta-\delta)}dy_0 \\ &= A + \theta^* e^{-\theta^* y}\tau\beta\int_0^y e^{-y_0(\theta+\tau-\theta^*)}dy_0 + \theta^* e^{-\theta^* y}(1-\beta)\delta\int_0^y e^{-y_0(\theta+\delta-\theta^*)}dy_0 \\ &= A + \theta^* e^{-\theta^* y}\tau\beta\left[\frac{e^{-y_0(\theta+\tau-\theta^*)}}{-(\theta+\tau-\theta^*)}\right]_0^y + \theta^* e^{-\theta^* y}(1-\beta)\delta\left[\frac{e^{-y_0(\theta+\delta-\theta^*)}}{-(\theta+\delta-\theta^*)}\right]_0^y \end{aligned}$$

$$= A + \frac{\theta^* e^{-\theta^* y} \tau\beta}{(\theta + \tau - \theta^*)}\left[1 - e^{-y(\theta+\tau-\theta^*)}\right] + \frac{\theta^* e^{-\theta^* y}(1-\beta)\delta}{(\theta + \delta - \theta^*)}\left[1 - e^{-y(\theta+\delta-\theta^*)}\right]$$

where A = $\theta\, e^{-\theta y}\left(\beta e^{-\tau y} + (1-\beta)e^{-\delta y}\right)$

$$\therefore K(y) = A + \theta^* e^{-\theta^* y}\left\{\left\{\frac{\beta\tau\left(1 - e^{-y(\theta+\tau-\theta^*)}\right)}{(\theta+\tau-\theta^*)}\right\} + \frac{(1-\beta)\delta}{(\theta+\delta-\theta^*)}\left\{1 - e^{-y(\theta+\delta-\theta^*)}\right\}\right\}$$

Equation (3) becomes

$$\frac{d}{h+d} = \sum_{n=1}^{\infty} F_n(B)\left\{\int_0^{\infty}[G_n(Y) - G_{n+1}(Y)]\left[\theta\, e^{-\theta y}\left(\beta e^{-\tau y} + (1-\beta)e^{-\delta y}\right) + \theta^* e^{-\theta^* y}\left\{\left\{\frac{\beta\,\tau\left(1 - e^{-y(\theta+\tau-\theta^*)}\right)}{(\theta+\tau-\theta^*)}\right\} + \frac{(1-\beta)\delta}{(\theta+\delta-\theta^*)}\left\{1 - e^{-y(\theta+\delta-\theta^*)}\right\}\right\}\right] dy\right\} \quad (3.4)$$

$$\frac{d}{h+d} = \sum_{n=1}^{\infty} F_n(B)\left\{\int_0^{\infty}[G_n(Y) - G_{n+1}(Y)]\left[\theta\, e^{-\theta y}\left(\beta e^{-\tau y} + (1-\beta)e^{-\delta y}\right) + \theta^* e^{-\theta^* y}\left\{\left\{\frac{\beta\,\tau\left(1 - e^{-y(\theta+\tau-\theta^*)}\right)}{(\theta+\tau-\theta^*)}\right\} + \frac{(1-\beta)\delta}{(\theta+\delta-\theta^*)}\left\{1 - e^{-y(\theta+\delta-\theta^*)}\right\}\right\}\right] dy\right\}$$

Since U_i follows Exponential distribution with parameter λ, $G_n(y)$ is the distribution function of $U_1 + U_2 + U_3 + \ldots\ldots + U_n$. Hence $G_n(y)$ is the n - fold convolution G(.) and thus $G_n(y)$ is the distribution function of Gamma distribution.

i.e., $G_n(y) = 1 - \sum_{i=0}^{n-1} \frac{[\lambda y]^i}{n!} e^{-\lambda y}$

$$G_{n+1}(y) = 1 - \sum_{i=0}^{n} \frac{[\lambda y]^i}{n!} e^{-\lambda y}$$

$$\therefore G_n(y) - G_{n+1}(y) = \frac{[\lambda y]^n}{n!} e^{-\lambda y}$$

$$\frac{d}{h+d} = \sum_{n=1}^{\infty}\int_0^{\infty} F_n(B)\frac{[\lambda y]^n}{n!} e^{-\lambda y}\left[\theta e^{-\theta y}\left[\beta e^{-\tau y} + (1-\beta)e^{-\delta y}\right] + \theta^* e^{-\theta^* y}\left\{\left[\frac{\beta\,\tau\left(1 - e^{-y(\theta+\tau-\theta^*)}\right)}{(\theta+\tau-\theta^*)}\right] + \frac{(1-\beta)\delta}{(\theta+\delta-\theta^*)}\left[1 - e^{-y(\theta+\delta-\theta^*)}\right]\right\}\right] dy$$

$$= \sum_{n=1}^{\infty} F_n(B)\left\{\int_0^{\infty}\frac{[\lambda y]^n}{n!} e^{-\lambda y}\theta\beta e^{-\theta y} e^{-\tau y} dy + \int_0^{\infty}\frac{[\lambda y]^n}{n!} e^{-\lambda y}\theta(1-\beta) e^{-\theta y} e^{-\delta y} dy + \int_0^{\infty}\frac{[\lambda y]^n}{n!}\frac{e^{-\lambda y}\theta^*\beta\,\tau e^{-\theta^* y}}{(\theta+\tau-\theta^*)} dy - \int_0^{\infty}\frac{[\lambda y]^n}{n!}\frac{e^{-\lambda y}\theta^*\beta e^{-\theta^* y}\,\tau e^{-y(\theta+\tau-\theta^*)}}{(\theta+\tau-\theta^*)} dy + \frac{(1-\beta)\delta\theta^*}{(\theta+\delta-\theta^*)}\int_0^{\infty}\frac{[\lambda y]^n}{n!} e^{-\lambda y} e^{-\theta^* y} dy - \frac{(1-\beta)\delta\theta^*}{(\theta+\delta-\theta^*)}\int_0^{\infty}\frac{[\lambda y]^n}{n!} e^{-\lambda y} e^{-\theta^* y} e^{-y(\theta+\delta-\theta^*)} dy\right\}$$

$$= \sum_{n=1}^{\infty} F_n(B)[I_1 + I_2 + I_3 - I_4 + I_5 - I_6] \quad (3.5)$$

Consider

$$I_1 = \frac{\lambda^n \theta\beta}{n!}\int_0^\infty y^n e^{-y(\lambda+\theta+\tau)}dy$$

$$= \frac{\lambda^n \theta\beta}{n!}\frac{\Gamma(n+1)}{(\lambda+\theta+\tau)^{n+1}}$$

$$= \frac{\lambda^n \theta\beta}{(\lambda+\theta+\tau)^{n+1}}$$

$$I_2 = \frac{\lambda^n \theta(1-\beta)}{n!}\int_0^\infty y^n e^{-y(\lambda+\theta+\delta)}dy$$

$$= \frac{\lambda^n (1-\beta)\theta}{(\lambda+\theta+\delta)^{n+1}}$$

$$I_3 = \frac{\lambda^n \theta^* \beta\,\tau}{n!\,(\theta+\tau-\theta^*)}\int_0^\infty y^n e^{-y(\lambda+\theta^*)}dy$$

$$= \frac{\lambda^n \theta^* \beta\,\tau}{(\theta+\tau-\theta^*)(\lambda+\theta^*)^{n+1}}$$

$$I_4 = \frac{\lambda^n \theta^* \beta\tau}{n!\,(\theta+\tau-\theta^*)}\int_0^\infty y^n e^{-y(\lambda+\theta+\tau)}dy$$

$$= \frac{\lambda^n \theta^* \beta\tau}{(\theta+\tau-\theta^*)(\lambda+\theta+\tau)^{n+1}}$$

$$I_5 = \frac{(1-\beta)\delta\theta^*\lambda^n}{n!\,(\theta+\delta-\theta^*)}\int_0^\infty y^n e^{-y(\lambda+\theta^*)}dy$$

$$= \frac{\lambda^n \theta^*(1-\beta)\delta}{(\theta+\delta-\theta^*)(\lambda+\theta^*)^{n+1}}$$

$$I_6 = \frac{(1-\beta)\delta\theta^*\lambda^n}{n!\,(\theta+\delta-\theta^*)}\int_0^\infty y^n e^{-y(\lambda+\delta+\theta)}dy$$

$$= \frac{\lambda^n \theta^*(1-\beta)\delta}{(\theta+\delta-\theta^*)(\lambda+\theta+\delta)^{n+1}}$$

$$\frac{d}{h+d} = \sum_{n=1}^{\infty} F_n(B)\Bigg[\frac{\lambda^n\theta\beta}{(\lambda+\theta+\tau)^{n+1}} + \frac{\lambda^n(1-\beta)\theta}{(\lambda+\theta+\delta)^{n+1}} + \frac{\lambda^n\theta^*\beta\tau}{(\theta+\tau-\theta^*)(\lambda+\theta^*)^{n+1}} - \frac{\lambda^n\theta^*\beta\tau}{(\theta+\tau-\theta^*)(\lambda+\theta+\tau)^{n+1}} + \frac{\lambda^n\theta^*(1-\beta)\delta}{(\theta+\delta-\theta^*)(\lambda+\theta^*)^{n+1}} - \frac{\lambda^n\theta^*(1-\beta)\delta}{(\theta+\delta-\theta^*)(\lambda+\theta+\delta)^{n+1}}\Bigg]$$

where

$$F_n(B) = \int_0^B \frac{\mu^n}{\Gamma n} y^{n-1} e^{-\mu y}dy$$

$$\frac{d}{h+d} = [T_1 + T_2 + T_3 - T_4 + T_5 - T_6] \tag{3.6}$$

Consider

$$T_1 = \sum_{n=1}^{\infty}\int_0^B \frac{\theta\beta(\lambda\mu)^n}{\Gamma n}\frac{y^{n-1}e^{-\mu y}}{(\lambda+\theta+\tau)^{n+1}}dy$$

$$= \theta\beta\int_0^B \sum_{n=1}^{\infty}\frac{(\lambda\mu)^n y^{n-1}}{(\lambda+\theta+\tau)^{n+1}}\frac{e^{-\mu y}}{\Gamma n}dy$$

$$= \frac{\lambda\theta\beta\mu}{(\lambda+\theta+\tau)^2}\int_0^B \sum_{n=1}^{\infty}\frac{\left(\frac{\lambda\mu y}{\lambda+\theta+\tau}\right)^{n-1}}{(n-1)!}e^{-\mu y}\ dy$$

$$T_1 = \frac{\lambda\theta\beta}{(\lambda+\theta+\tau)}\left[\frac{1-e^{-\mu B\left(\frac{\theta+\tau}{\lambda+\theta+\tau}\right)}}{(\theta+\tau)}\right]$$

Consider

$$T_2 = \sum_{n=1}^{\infty}\int_0^B \frac{\theta(1-\beta)(\lambda\mu)^n}{\Gamma n}\frac{y^{n-1}e^{-\mu y}}{(\lambda+\theta+\delta)^{n+1}}dy$$

$$= \theta(1-\beta)\int_0^B \sum_{n=1}^{\infty}\frac{(\lambda\mu)^n y^{n-1}}{(\lambda+\theta+\delta)^{n+1}}\frac{e^{-\mu y}}{\Gamma n}dy$$

$$=$$

$$T_2 = \frac{\lambda\theta(1-\beta)}{(\lambda+\theta+\delta)}\left[\frac{1-e^{-\mu B\left(\frac{\theta+\delta}{\lambda+\theta+\delta}\right)}}{(\theta+\delta)}\right]$$

Consider

$$T_3 = \sum_{n=1}^{\infty}\int_0^B \frac{\theta^*\tau\beta(\lambda\mu)^n}{\Gamma n}\frac{y^{n-1}e^{-\mu y}}{(\lambda+\theta^*)^{n+1}(\theta+\tau-\theta^*)}dy$$

$$= \theta^*\tau\beta\int_0^B \sum_{n=1}^{\infty}\frac{(\lambda\mu)^n y^{n-1}}{(\lambda+\theta^*)^{n+1}(\theta+\tau-\theta^*)}\frac{e^{-\mu y}}{\Gamma n}dy$$

$$T_3 = \frac{\lambda\tau\beta}{(\lambda+\theta^*)(\theta+\tau-\theta^*)}\left[1-e^{-\mu B\left(\frac{\theta^*}{\lambda+\theta^*}\right)}\right]$$

Consider

$$T_4 = \sum_{n=1}^{\infty}\int_0^B \frac{\theta^*\tau\beta(\lambda\mu)^n}{\Gamma n}\frac{y^{n-1}e^{-\mu y}}{(\lambda+\theta+\tau)^{n+1}(\theta+\tau-\theta^*)}dy$$

$$= \frac{\lambda\theta^*\tau\beta\mu}{(\lambda+\theta+\tau)^2(\theta+\tau-\theta^*)}\left[\frac{e^{-\mu y\left(\frac{\theta+\tau}{\lambda+\theta+\tau}\right)}}{-\mu\left(\frac{\theta+\tau}{\lambda+\theta+\tau}\right)}\right]_0^B$$

$$T_4 = \frac{\lambda\theta^*\tau\beta}{(\lambda+\theta+\tau)(\theta+\tau-\theta^*)(\theta+\tau)}\left[1-e^{-\mu B\left(\frac{\theta+\tau}{\lambda+\theta+\tau}\right)}\right]$$

Similarly

$$T_5 = \frac{\lambda\delta(1-\beta)}{(\lambda+\theta^*)(\theta+\delta-\theta^*)}\left[1-e^{-\mu B\left(\frac{\theta^*}{\lambda+\theta^*}\right)}\right]$$

and

$$T_6 = \frac{\lambda\delta\theta^*(1-\beta)}{(\lambda+\theta+\delta)(\theta+\delta-\theta^*)(\theta+\delta)}\left[1-e^{-\mu B\left(\frac{\theta+\delta}{\lambda+\theta+\delta}\right)}\right]$$

$$\begin{aligned}\frac{d}{h+d} = \Bigg[&\left\{\frac{\lambda\theta\beta}{(\lambda+\theta+\tau)}\left[\frac{1-e^{-\mu B\left(\frac{\theta+\tau}{\lambda+\theta+\tau}\right)}}{(\theta+\tau)}\right]\right\} + \left\{\frac{\lambda\theta(1-\beta)}{(\lambda+\theta+\delta)}\left[\frac{1-e^{-\mu B\left(\frac{\theta+\delta}{\lambda+\theta+\delta}\right)}}{(\theta+\delta)}\right]\right\}\\
&+\left\{\frac{\lambda\tau\beta}{(\lambda+\theta^*)(\theta+\tau-\theta^*)}\left[1-e^{-\mu B\left(\frac{\theta^*}{\lambda+\theta^*}\right)}\right]\right\}\\
&-\left\{\frac{\lambda\theta^*\tau\beta}{(\lambda+\theta+\tau)(\theta+\tau-\theta^*)(\theta+\tau)}\left[1-e^{-\mu B\left(\frac{\theta+\tau}{\lambda+\theta+\tau}\right)}\right]\right\}\\
&+\left\{\frac{\lambda\delta(1-\beta)}{(\lambda+\theta^*)(\theta+\delta-\theta^*)}\left[1-e^{-\mu B\left(\frac{\theta^*}{\lambda+\theta^*}\right)}\right]\right\}\\
&-\left\{\frac{\lambda\delta\theta^*(1-\beta)}{(\lambda+\theta+\delta)(\theta+\delta-\theta^*)(\theta+\delta)}\left[1-e^{-\mu B\left(\frac{\theta+\delta}{\lambda+\theta+\delta}\right)}\right]\right\}\Bigg]\end{aligned}$$

Thus,

$$\frac{d}{h+d}=\left\{\frac{\lambda\beta\left[1-e^{-\mu B\left(\frac{\theta+\tau}{\lambda+\theta+\tau}\right)}\right]}{(\theta+\tau)(\lambda+\theta+\tau)}\right\}\left\{\theta-\frac{\theta^*\tau}{(\theta+\tau-\theta^*)}\right\}+\left\{\frac{\lambda(1-\beta)\left[1-e^{-\mu B\left(\frac{\theta+\delta}{\lambda+\theta+\delta}\right)}\right]}{(\theta+\delta)(\lambda+\theta+\delta)}\right\}\left\{\theta-\frac{\delta\theta^*}{(\theta+\delta-\theta^*)}\right\}+\left\{\frac{\lambda\left[1-e^{-\mu B\left(\frac{\theta^*}{\lambda+\theta^*}\right)}\right]}{(\lambda+\theta^*)}\right\}\left\{\frac{\tau\beta}{(\theta+\tau-\theta^*)}+\frac{\delta(1-\beta)}{(\theta+\delta-\theta^*)}\right\} \quad (3.7)$$

The above derived equation for optimal Base Stock can be stated as the following theorem.

Theorem *The optimal Base Stock $\hat{B}$ can be obtained in the equation 3.7 when the probability density function of continuous lead time random variable satisfies SCBZ property under the assumption that the truncation point is also a random variable which follows mixed exponential distribution with parameter τ and δ.*

Corollary *If the pdf of continuous lead time random variable is exponential and which satisfies the SCBZ property under the assumptions that the truncation point is also a random variable which follows mixed exponential distribution with parameter τ and δ.*
Then the optimal base stock $\hat{B}$ is

$$\frac{d}{h+d}=\frac{\lambda}{(\lambda+\tau+\theta)(\tau+\theta)}\left[\theta-\frac{\tau\theta^*}{(\tau+\theta-\theta^*)}\right]\left[1-e^{-\mu B\left(\frac{\theta+\tau}{\lambda+\theta+\tau}\right)}\right]+\frac{\lambda\tau\left[1-e^{-\frac{\theta^*\mu B}{\lambda+\theta^*}}\right]}{(\lambda+\theta-\theta^*)(\lambda+\theta^*)}$$

The above result can also be obtained from the equation (3.7), When $\beta = 1$.

4. PROPERTIES OF OPTIMAL BASE STOCK

Let the equation 3.7 be denoted as Δ (.).

4.1. *Case (i)*

The variations of optimal value of B ̂for the changes in the values of 'h' is

$$\Delta(h)=\frac{d}{h+d}-\left\{\frac{\lambda\beta\left[1-e^{-\mu B\left(\frac{\theta+\tau}{\lambda+\theta+\tau}\right)}\right]}{(\theta+\tau)(\lambda+\theta+\tau)}\right\}\left\{\theta-\frac{\theta^*\tau}{(\theta+\tau-\theta^*)}\right\}+\left\{\frac{\lambda(1-\beta)\left[1-e^{-\mu B\left(\frac{\theta+\delta}{\lambda+\theta+\delta}\right)}\right]}{(\theta+\delta)(\lambda+\theta+\delta)}\right\}\left\{\theta-\frac{\delta\theta^*}{(\theta+\delta-\theta^*)}\right\}+\left\{\frac{\lambda\left[1-e^{-\mu B\left(\frac{\theta^*}{\lambda+\theta^*}\right)}\right]}{(\lambda+\theta^*)}\right\}\left\{\frac{\tau\beta}{(\theta+\tau-\theta^*)}+\frac{\delta(1-\beta)}{(\theta+\delta-\theta^*)}\right\}$$

And

$$\Delta(h)=\frac{d}{h_1+d}-\left\{\frac{\lambda\beta\left[1-e^{-\mu B\left(\frac{\theta+\tau}{\lambda+\theta+\tau}\right)}\right]}{(\theta+\tau)(\lambda+\theta+\tau)}\right\}\left\{\theta-\frac{\theta^*\tau}{(\theta+\tau-\theta^*)}\right\}+\left\{\frac{\lambda(1-\beta)\left[1-e^{-\mu B\left(\frac{\theta+\delta}{\lambda+\theta+\delta}\right)}\right]}{(\theta+\delta)(\lambda+\theta+\delta)}\right\}\left\{\theta-\frac{\delta\theta^*}{(\theta+\delta-\theta^*)}\right\}+\left\{\frac{\lambda\left[1-e^{-\mu B\left(\frac{\theta^*}{\lambda+\theta^*}\right)}\right]}{(\lambda+\theta^*)}\right\}\left\{\frac{\tau\beta}{(\theta+\tau-\theta^*)}+\frac{\delta(1-\beta)}{(\theta+\delta-\theta^*)}\right\}$$

Therefore $\Delta(h)-\Delta(h_1)=\frac{d}{h+d}-\frac{d}{h1+d}$

Since $h<h_1$, $\Delta(h)-\Delta(h_1)>0$
Which implies that $\Delta(h)>\Delta(h_1)$, for all h < $h_1$of holding cost.

For the fixed values of h, d, λ, β, μ, θ, θ^*, τ and δ, the optimal value of B, can be obtained.

For d=3, λ=2, β=0.5, τ=1, δ=1.5, θ*=1, μ=1.5, the optimal value of B is obtained and the variations in $\hat{B}$ for the changes in the value of h.

h	5	10	15	20
B	1.6207	0.877	0.604	0.461

4.2. ***Case (ii)***

The variations of optimal value of $\hat{B}$ for the changes in the values of 'd' is

$$\Delta(h) = \frac{d}{h+d} - \left\{\frac{\lambda\beta\left[1-e^{-\mu B\left(\frac{\theta+\tau}{\lambda+\theta+\tau}\right)}\right]}{(\theta+\tau)(\lambda+\theta+\tau)}\right\}\left\{\theta - \frac{\theta^*\tau}{(\theta+\tau-\theta^*)}\right\} + \left\{\frac{\lambda(1-\beta)\left[1-e^{-\mu B\left(\frac{\theta+\delta}{\lambda+\theta+\delta}\right)}\right]}{(\theta+\delta)(\lambda+\theta+\delta)}\right\}\left\{\theta - \frac{\delta\theta^*}{(\theta+\delta-\theta^*)}\right\} + \left\{\frac{\lambda\left[1-e^{-\mu B\left(\frac{\theta^*}{\lambda+\theta^*}\right)}\right]}{(\lambda+\theta^*)}\right\}\left\{\frac{\tau\beta}{(\theta+\tau-\theta^*)} + \frac{\delta(1-\beta)}{(\theta+\delta-\theta^*)}\right\}$$

And

$$\Delta(d_1) = \frac{d_1}{h+d_1} - \left\{\frac{\lambda\beta\left[1-e^{-\mu B\left(\frac{\theta+\tau}{\lambda+\theta+\tau}\right)}\right]}{(\theta+\tau)(\lambda+\theta+\tau)}\right\}\left\{\theta - \frac{\theta^*\tau}{(\theta+\tau-\theta^*)}\right\} + \left\{\frac{\lambda(1-\beta)\left[1-e^{-\mu B\left(\frac{\theta+\delta}{\lambda+\theta+\delta}\right)}\right]}{(\theta+\delta)(\lambda+\theta+\delta)}\right\}\left\{\theta - \frac{\delta\theta^*}{(\theta+\delta-\theta^*)}\right\} + \left\{\frac{\lambda\left[1-e^{-\mu B\left(\frac{\theta^*}{\lambda+\theta^*}\right)}\right]}{(\lambda+\theta^*)}\right\}\left\{\frac{\tau\beta}{(\theta+\tau-\theta^*)} + \frac{\delta(1-\beta)}{(\theta+\delta-\theta^*)}\right\}$$

Therefore $\Delta(d) - \Delta(d_1) = \frac{d_1}{h+d_1} - \frac{d_1}{h+d_1}$

$$= \frac{(h+d_1)d - d_1(h+d)}{(h+d_1)(h+d)}$$

$$\Delta(d) - \Delta(d_1) = \frac{h(d-d_1)}{(h+d_1)(h+d)}, d < d_1$$

$$\Delta(d) < \Delta(d_1), \qquad when\, d < d_1$$

That is when the shortage cost increases, the optimal Base Stock also increases.
For h=5, λ=2, β=0.5, τ=1, δ=1.5, θ*=1, μ=1.5, the optimal value of B is obtained and the variations in $\hat{B}$ for the changes in the value of d.

d	3	5	7	9
B	1.6207	2.516	3.383	4.297

4.3. ***Case (iii)***

The variations of optimal value of $\hat{B}$for the changes in the values of 'λ' is

$$\Delta(\lambda)-\Delta(\lambda_1)=\frac{\beta}{(\theta+\tau)}\left\{\theta-\frac{\theta^*\tau}{(\theta+\tau-\theta^*)}\right\}\left\{\frac{\lambda\left[1-e^{-\mu B\left(\frac{\theta+\tau}{\lambda+\theta+\tau}\right)}\right]}{(\lambda+\theta+\tau)}-\frac{\lambda_1\left[1-e^{-\mu B\left(\frac{\theta+\tau}{\lambda_1+\theta+\tau}\right)}\right]}{(\lambda_1+\theta+\tau)}\right\}$$
$$+\left\{\theta-\frac{\delta\theta^*}{(\theta+\delta-\theta^*)}\right\}\frac{(1-\beta)}{(\theta+\delta)}\left\{\frac{\lambda\left[1-e^{-\mu B\left(\frac{\theta+\delta}{\lambda+\theta+\delta}\right)}\right]}{(\lambda+\theta+\delta)}-\frac{\lambda_1\left[1-e^{-\mu B\left(\frac{\theta+\delta}{\lambda_1+\theta+\delta}\right)}\right]}{(\lambda_1+\theta+\delta)}\right\}$$
$$+\left\{\frac{\tau\beta}{(\theta+\tau-\theta^*)}+\frac{\delta(1-\beta)}{(\theta+\delta-\theta^*)}\right\}\left\{\frac{\lambda\left[1-e^{-\mu B\left(\frac{\theta^*}{\lambda+\theta^*}\right)}\right]}{(\lambda+\theta^*)}-\frac{\lambda_1\left[1-e^{-\mu B\left(\frac{\theta^*}{\lambda_1+\theta^*}\right)}\right]}{(\lambda_1+\theta^*)}\right\}$$

$$\Delta(\lambda)-\Delta(\lambda_1)=\frac{\beta}{(\theta+\tau)}\left\{\theta-\frac{\theta^*\tau}{(\theta+\tau-\theta^*)}\right\}\left\{\frac{\lambda\left[1-e^{-f(\lambda)}\right]}{(\lambda+\theta+\tau)}-\frac{\lambda_1\left[1-e^{-f(\lambda_1)}\right]}{(\lambda_1+\theta+\tau)}\right\}$$
$$+\left\{\theta-\frac{\delta\theta^*}{(\theta+\delta-\theta^*)}\right\}\frac{(1-\beta)}{(\theta+\delta)}\left\{\frac{\lambda\left[1-e^{-f(\lambda)}\right]}{(\lambda+\theta+\delta)}-\frac{\lambda_1\left[1-e^{-f(\lambda_1)}\right]}{(\lambda_1+\theta+\delta)}\right\}$$
$$+\left\{\frac{\tau\beta}{(\theta+\tau-\theta^*)}+\frac{\delta(1-\beta)}{(\theta+\delta-\theta^*)}\right\}\left\{\frac{\lambda\left[1-e^{-f(\lambda)}\right]}{(\lambda+\theta^*)}-\frac{\lambda_1\left[1-e^{-f(\lambda_1)}\right]}{(\lambda_1+\theta^*)}\right\}$$

Since e^{-x} is decreasing function, hence $e^{-f(\lambda_1)}<e^{-f(\lambda)}$, thus

$\Delta(\lambda)-\Delta(\lambda_1)<0$, for all $\lambda<\lambda_1$.

For h=5, d=3, β=0.5, τ=1, δ=1.5, θ* = 1, μ =1.5, the optimal value of B is obtained and the variations in $\hat{B}$ for the changes in the value of λ.

$\boldsymbol{\lambda}$	2	2.5	3	3.5
B	1.6207	1.793	1.986	2.191

4.4. ***Case (iv)***

The variations of optimal value of $\hat{B}$for the changes in the values of 'μ' is

$$\Delta(\mu)-\Delta(\mu_1)=\left\{\frac{\lambda\beta}{(\theta+\tau)(\lambda+\theta+\tau)}\right\}\left\{\theta-\frac{\theta^*\tau}{(\theta+\tau-\theta^*)}\right\}\left[1-e^{-\mu B\left(\frac{\theta+\tau}{\lambda+\theta+\tau}\right)}-\left[1-e^{-\mu_1 B\left(\frac{\theta+\tau}{\lambda+\theta+\tau}\right)}\right]\right]$$
$$+\left\{\frac{\lambda(1-\beta)}{(\theta+\delta)(\lambda+\theta+\delta)}\right\}\left\{\theta-\frac{\delta\theta^*}{(\theta+\delta-\theta^*)}\right\}\left[\left[1-e^{-\mu B\left(\frac{\theta+\delta}{\lambda+\theta+\delta}\right)}\right]-\left[1-e^{-\mu_1 B\left(\frac{\theta+\delta}{\lambda+\theta+\delta}\right)}\right]\right]$$
$$+\left\{\frac{\lambda\left[1-e^{-\mu B\left(\frac{\theta^*}{\lambda+\theta^*}\right)}\right]}{(\lambda+\theta^*)}\right\}\left\{\frac{\tau\beta}{(\theta+\tau-\theta^*)}+\frac{\delta(1-\beta)}{(\theta+\delta-\theta^*)}\right\}\left[\left[1-e^{-\mu B\left(\frac{\theta^*}{\lambda+\theta^*}\right)}\right]-\left[1-e^{-\mu_1 B\left(\frac{\theta^*}{\lambda+\theta^*}\right)}\right]\right]$$

Since e^{-x} is decreasing function , hence $e^{-f(\mu_1)} < e^{-f(\mu)}$, thus

$$\Delta(\mu)-\Delta(\mu_1)<0$$
$$\Delta(\mu)>\Delta(\mu_1)$$

For h=5, d=3, λ=2, β=0.5, τ=1, δ=1.5, θ*=1, the optimal value of B is obtained and the variations in $\hat{B}$ for the changes in the value of μ.

μ	1.5	2	2.5	3
B	1.6207	1.216	0.972	0.81

5. CONCLUSION

From the tables and graphs, it is observed that,

As the holding cost (h) increases, the optimal Base Stock $\hat{B}$ decreases.

As the shortage cost (d) increases, the optimal Base Stock $\hat{B}$ increases.

As λ, the parameter of inter arrival time distribution increases, the optimal Base Stock $\hat{B}$ decreases.

As μ, the parameter of demand distribution increases, the optimal Base Stock $\hat{B}$ increases.

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